



RAN - 2103000204023001

RAN-2103000204023001 / 2103000204033001 / 2111000304023001 /  
2111000304033001

S. Y. B. Sc. (Sem. - IV) Examination September - 2023

MTH-401 : Mathematics - VIII

Time: 2 Hours ]

[ Total Marks: 50

सूचना : / Instructions

(1)

नीचे दशावलि निशानीवाणी विगतो उत्तरवली पर अवश्य लपवी.  
Fill up strictly the details of signs on your answer book

Name of the Examination:

S. Y. B. Sc. (Sem. - IV)

Name of the Subject :

MTH-401 : Mathematics - VIII

Subject Code No.: 2103000204023001 / 2103000204033001 / 2111000304023001 /  
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Seat No.:

Student's Signature

- (2) All questions are compulsory.  
(3) Figures to the right indicate marks of the questions.  
(4) Follow usual notations and conventions.

Q. 1. Answer any FIVE from the following questions.

[10]

- (1) Define : Beta and Gamma Function.  
(2) Show that  $\beta(l+1, m) + \beta(l, m+1) = \beta(l, m)$ .

(3) Evaluate :  $\int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{\pi} \cos(x+y) dx dy$ .

(4) Evaluate :  $\int_1^2 \int_0^{\frac{y}{2}} y dy dx$ .

(5) Find  $L\{\cos at\}$ .

(6) Find  $L\{3 \cosh 5t - 4 \sinh 5t\}$ .

(7) State the second shifting theorem for Laplace transform and inverse Laplace transform.

(8) Evaluate :  $L^{-1}\left\{\frac{2p+1}{p(p+1)}\right\}$ .

**Q. 2. Attempt any TWO. [10]**

(1) State and prove the relation between Beta and Gamma function.

(2) Show that :  $\int_0^{\infty} \frac{dx}{1+x^4} = \frac{\pi}{2\sqrt{2}}$ .

(3) Prove that :  $\left\{ \int_0^{\frac{\pi}{2}} \sin^n x dx \right\} \left\{ \int_0^{\frac{\pi}{2}} \sin^{n+1} x dx \right\} = \frac{\pi}{2(n+1)}$

**Q. 3. Attempt any TWO. [10]**

(1) Find the area enclosed between the parabolas  $y^2 = 4ax$  and  $x^2 = 4ay$  using double integral.

(2) If  $S$  is a triangular region bounded by the line  $y = 3x$ ,  $x$ -axis and the line  $x = 6$ , then Evaluate :  $\iint_S x^2 y^2 dx dy$ .

(3) Change the order of integration of the double integral  $\int_0^{\frac{y}{4}} \int_0^{\frac{y}{2}} f(x, y) dy dx$ .

**Q. 4. Attempt any TWO. [10]**

(1) State and prove the change of scale property for Laplace transform.

(2) Find  $L[\sin\sqrt{t}]$ .

(3) Evaluate :  $\int_0^{\infty} \frac{e^{-at} - e^{-bt}}{t} dt$

**Q. 5. Attempt any TWO. [10]**

(1) State and prove the linearity property for Inverse Laplace transform.

(2) Evaluate :  $L^{-1} \left[ \frac{3p + 2}{4p^2 + 12p + 9} \right]$

(3) Prove that if,

$$L^{-1} \left[ \frac{p}{(p^2 + 1)^2} \right] = \left( \frac{t}{2} \right) \sin t, \text{ then } L^{-1} \left[ \frac{32p}{(16p^2 + 1)^2} \right] = \left( \frac{t}{4} \right) \sin \left( \frac{t}{4} \right).$$